Eur J Appl Physiol (1998) 77: 492–497

ORIGINAL ARTICLE

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The mathematics of breaking away and chasing in cycling

Accepted: 29 October 1997

Abstract In cycling stage races a small group of riders will often form a "breakaway" and establish a lead over the main group. This paper examines the factors that affect the likelihood of success for the breakaway. A mathematical approach is used, drawing on a model of cycling previously developed and validated (Olds et al. J Appl Physiol 78:1596–1611, 1995). In a breakaway group, the power required to overcome air resistance is reduced because the lead can be shared, with trailing riders sheltering or drafting behind leading riders. The benefit of drafting can be quantified as a function of the distance between riders using previously obtained data. Of course, this advantage is even greater in the (larger) chasing group, so that eventually the chasing group will catch the breakaway, assuming identical bicycles and physiological characteristics. The question addressed is: what factors determine how great a lead the breakaway must have in order for the chasing group to be unable to catch the breakaway before the finish of the race? Demand-side simulations show that the critical factors are: the distance remaining in the race; the speed of the breakaway group; the number of riders in the chasing and breakaway groups; how closely riders in each group draft one another; the grade; surface roughness; as well as head- and cross-winds. When supply-side physiological factors are incorporated, the maximum sustainable speed and maximum lead time can be calculated.

Key words Bicycle racing · Breakaways · Drafting · Mathematical model

Introduction

In stage races such as the Tour de France it is common to see a group of riders form a "breakaway" ahead of the main group, or peloton. Riders not in the breakaway must decide at what point to attack and try to catch the breakaway group, so as not to let them gain a time advantage. Sometimes these breakaway groups are successful, and reach the finishing line clear of the peloton. More frequently, they are caught by the peloton. This paper takes a mathematical modelling approach to quantify factors that determine the success or failure of a breakaway. It uses a previously developed mathematical model of cycling (Olds et al. 1993, 1995) to quantify the effects on the likelihood of breakaway success of demand-side variables (e.g. the relative numbers of riders in the breakaway and chasing groups, the spacing between riders, the speed of the breakaway, the lead they have established over the chasing group, and the terrain and environmental conditions) and supply-side variables (maximal oxygen consumption or $\dot{V}O_{2\,max}$ and the relationship between time to exhaustion and the sustainable fraction of $\dot{V}O_{2\,max}$). Although the assumptions of the model represent a somewhat idealized situation, it is possible that the model can be used in practical circumstances to make decisions about whether, when and how to break away and to chase.

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Methods

Reduction in air resistance due to drafting

In competitive cycling on the flat, air resistance is by far the greatest force opposing the forward motion of the cyclist. Air resistance can be dramatically reduced by riding in the slipstream of another rider or vehicle. The following rider will then enjoy the low pressure area behind the lead rider. Drafting is illegal in some forms of cycle racing, but is legal and commonly practised in track races, stage races and team time-trials. It is therefore useful to have some means of quantifying the effects of drafting.

Kyle (1979) experimentally measured the effect of drafting on wind resistance using the rolldown technique. He found that the reduction in air resistance diminishes as the wheel spacing (d_w, m) increases in a paceline. Kyle mentions that the reduction in air resistance for cyclists in the racing position is well described by a second-order polynomial, but does not report the equation. From graphical data (his Fig. 2), the following equation has been constructed:

$$\mathrm{CF_{draft}} = 0.62 - 0.0104\, d_{\mathrm{w}} + 0.0452\, d_{\mathrm{w}}^2 \tag{1}$$

where $\mathrm{CF_{draft}}$ is a correction factor, the ratio of the resistance under drafting conditions to that under conditions where no drafting occurs, and d_w is the wheel-to-wheel distance (m) between the bicycle and the preceding rider. For $d_w \geq 3$ m, $\mathrm{CF_{draft}}$ is assumed to be 1 – that is, there is no benefit to be had by drafting more than 3 m behind another cyclist.

Let the external power required for the bicycle-rider system to overcome air resistance be ${}^{\mathrm{ext}}P_{\mathrm{air}}$ (in W), which under windless conditions is proportional to the cube of the velocity of the system (v, $\mathrm{m \cdot s^{-1}}$), i.e. ${}^{\mathrm{ext}}P_{\mathrm{air}} = kv^3$. We can then correct for drafting by multiplying ${}^{\mathrm{ext}}P_{\mathrm{air}}$ by $\mathrm{CF_{draft}}$:

$${}^{\mathrm{ext}}P_{\mathrm{air}} = k\, \mathrm{CF_{draft}}\, v^3 \tag{2}$$

where k is a lumped parameter, incorporating air density and projected frontal area. The correction factor applies no matter how many bicycles are in the preceding paceline, and regardless of the position of the cyclist in the paceline (Kyle 1979). Figure 1 shows the relationship between wheel spacing and changes in air resistance and $\dot{V}O_2$ (estimated using the model of Olds et al. 1995) for cyclists riding at 40 $\mathrm{km \cdot h^{-1}}$.

The effect of riding in groups

Both on the track and on the road, cyclists often ride in groups, alternating between leading a paceline and slipstreaming the lead rider(s) in a paceline. In this way the overall external power requirement of cycling is reduced in that at any one time at least some riders are taking advantage of the effect of drafting. It is relatively easy to estimate the reduction in the overall average power requirement when riding in a group in this manner at a constant speed (v, $\mathrm{m \cdot s^{-1}}$). Let there be n riders in a group, who ride close enough to the rider ahead that the drafting factor is $\mathrm{CF_{draft}}$. We will assume that these riders are identical. As they ride along, they must overcome air resistance and rolling resistance and they must also do work to ride up (or down) a grade. We will discount the power required to overcome bearing and mechanical friction, and assume that they are neither accelerating nor decelerating, and that there is no wind. To ride individually at a steady speed they require an external power output of ${}^{\mathrm{ext}}P_{\mathrm{air}} + {}^{\mathrm{ext}}P_{\mathrm{roll}} + {}^{\mathrm{ext}}P_{\mathrm{grade}}$ (in W), where ${}^{\mathrm{ext}}P_{\mathrm{roll}}$ is the external power required to overcome rolling resistance, and ${}^{\mathrm{ext}}P_{\mathrm{grade}}$ is the external power required to ride up (or down) a grade. Let the group travel for a period of t_f (in s). If these riders take equal turns at the front, each rider will spend $t_f n^{-1}$ (in s) at the front, and $t_f(n-1)n^{-1}$ (in s) either dropping back or drafting in the paceline. When they are riding behind (and, we will assume, when they are dropping back), their required power output will fall to $\mathrm{CF_{draft}}\,{}^{\mathrm{ext}}P_{\mathrm{air}} + {}^{\mathrm{ext}}P_{\mathrm{roll}} + {}^{\mathrm{ext}}P_{\mathrm{grade}}$ (W). The mean power output will be ${}^{\mathrm{ext}}P_{\mathrm{air}}\{[\mathrm{CF_{draft}}(n-1)+1]n^{-1}\} + {}^{\mathrm{ext}}P_{\mathrm{roll}} + {}^{\mathrm{ext}}P_{\mathrm{grade}}$. Each of the terms of this equation is a function of velocity, so that for any given velocity, the mean power can be calculated. This analysis is similar to that of Kyle (1979), except that Kyle neglects the influence of rolling resistance, and the power required to ride up a grade.

Incorporating supply-side variables

So far, we have ignored supply-side variables. We can increase the sensitivity of the model by including supply-side (physiological) variables. Of particular interest is the relationship between the time to exhaustion and the sustainable power output. There is a distinct relationship between the sustainable fraction of $\dot{V}O_{2\,max}$ – or more precisely metabolic power production expressed as a fraction of $\dot{V}O_{2\,max}$ – and time to exhaustion. This relationship has variously been characterized using linear (di Prampero et al. 1986), semilog (Péronnet et al. 1987), exponential (Léger et al. 1984), hyperbolic and log-log models. Figure 2 shows a log-log plot of the cumulated results of 38 studies ($n = 782$ data points, using weighted means). There is a good relationship ($r = 0.92$, $P \leq 0.0001$) between the log of time to exhaustion and the log of sustainable fraction of $\dot{V}O_{2\,max}$. The best-fit equation is

$$\ln(T_{\mathrm{lim}}) = -6.351 \ln(\mathrm{f}\,\dot{V}O_{2\,max}) + 2.478 \tag{3}$$

where T_{lim} is the time to exhaustion (min) and $\mathrm{f}\,\dot{V}O_{2\,max}$ is the sustainable fraction of $\dot{V}O_{2\,max}$. We will use this relationship to

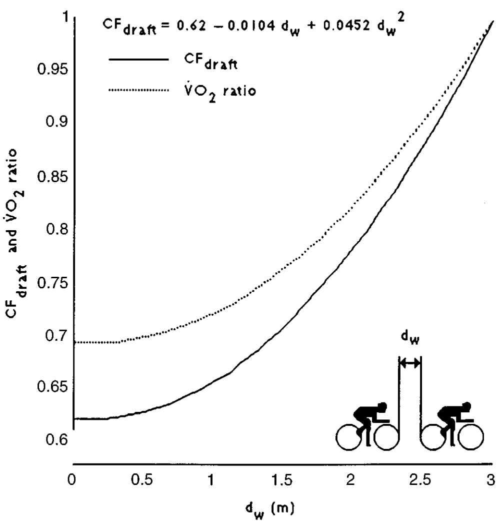


Fig. 1 Relationship between wheel spacing (d_w, in m) and a correction factor, $\mathrm{CF_{draft}}$ (the ratio of the resistance under drafting conditions to that under conditions where no drafting occurs) and the ratio of the oxygen consumption ($\dot{V}O_2$) required when drafting to the $\dot{V}O_2$ required for cycling without drafting ($\dot{V}O_2$ ratio). The simulations use the values reported by di Prampero et al. (1979) for cyclists riding at 40 $\mathrm{km \cdot h^{-1}}$

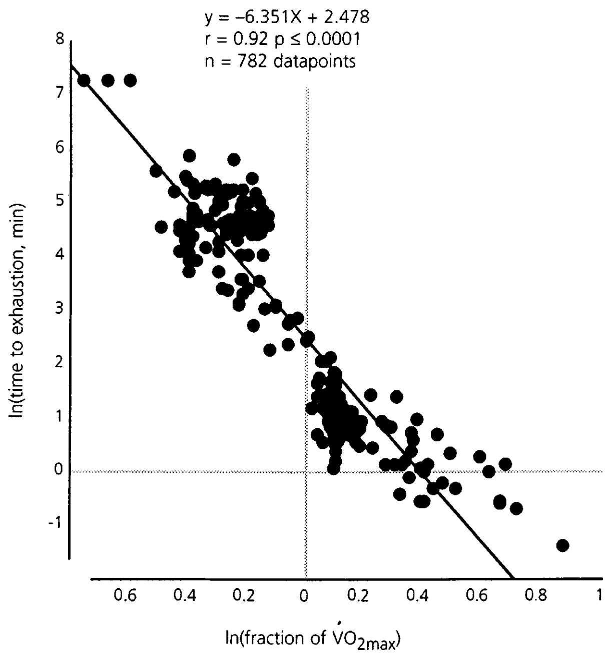


Fig. 2 Cumulated results from 38 studies ($n = 782$ data points) showing the relationship between the natural log of time to exhaustion and the natural log of the sustainable fraction of $\dot{V}O_{2\,max}$. The studies have used both cycle and treadmill exercise. The line of best fit is shown. Weighted means have been used

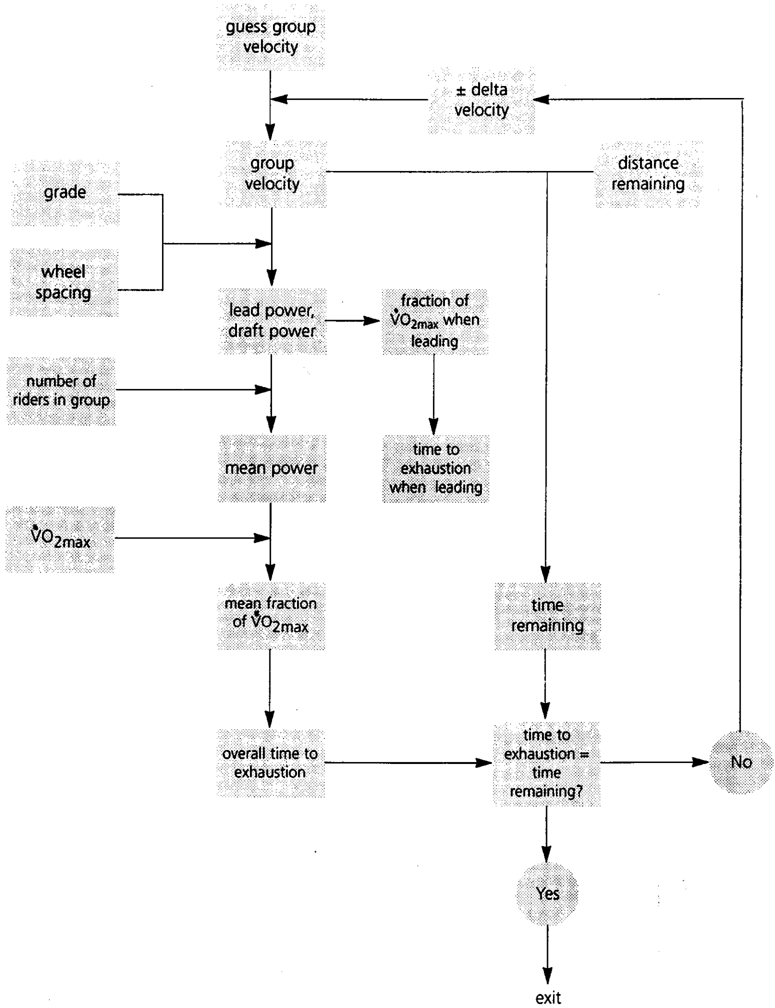


Fig. 3 Flowchart showing the modelling strategy used in this paper. For a more detailed explanation, see text

incorporate supply-side factors into the model developed here. This will allow us to calculate the speed that a group of riders (be it a breakaway or a chasing group) can maintain for any given remaining race distance, and the maximum time that any rider in the group can spend at the head of the paceline.

The overall modelling strategy is shown in Fig. 3:

- We start by guessing the velocity the group can maintain.
- We can then, given the distance remaining, calculate how long it will take to finish the race.
- We can also calculate the power required when leading, and the power required when drafting (using the equations described above). If we know the $\dot{V}O_{2\,max}$ of the riders in the group, the power required when leading can be expressed as a fraction of $\dot{V}O_{2\,max}$ (assuming typical $\dot{V}O_2$-workrate relationships; Olds et al. 1995). From this the time to exhaustion when leading can be calculated, using the log-log relationship shown in Fig. 2.
- By working out the mean power for the group, we can calculate the fraction of $\dot{V}O_{2\,max}$ at which they are working, and the time to exhaustion at that power output (here we assume that the sustainable fraction of $\dot{V}O_{2\,max}$ is independent of the pattern of power production).
- If the overall time to exhaustion does not match the time required to finish the race, we adjust the guessed velocity accordingly, and continue in this fashion until they are equal.

By applying this procedure to both breakaway and chasing groups (understanding that variables such as the number in each group and distance remaining will differ between groups), we can decide whether the chasing group will catch the breakaway.

Results

All analyses contained in this section refer to the following baseline situation:

- A breakaway group of five riders is chased by a group of 10 riders.
- Both groups maintain a wheel spacing of 0.5 m.
- The $\dot{V}O_{2\,max}$ of all riders in $5\ l \cdot min^{-1}$.
- There is 20 km remaining to ride.
- The terrain is flat.

The demand-side model has been parameterized using the values reported by di Prampero et al. (1979). The values for relevant variables will be independently varied, while the others are held constant. The analysis will focus on the effects on the mean group velocity, the maximum lead time for any rider in a group, and the critical lead a breakaway requires for success.

The effects of a number of variables on mean group velocity are demonstrated in Fig. 4. Mean velocity rises rapidly as the number in the group increases from one to five, but then tends to flatten out. Velocity declines slowly as wheel spacing increases from 0.1 to 0.5 m,

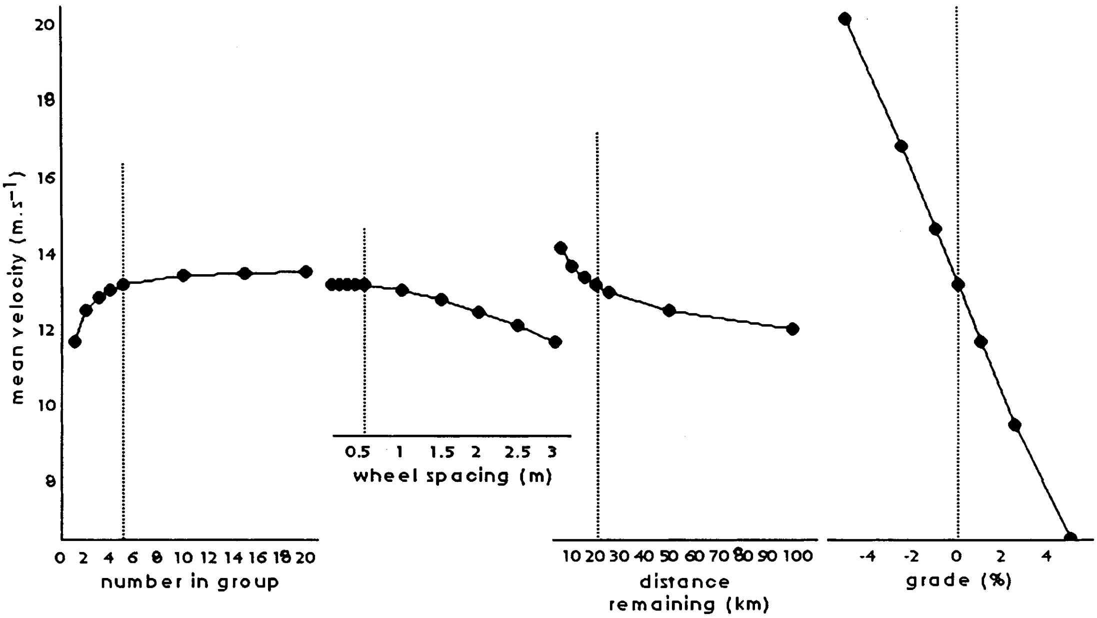


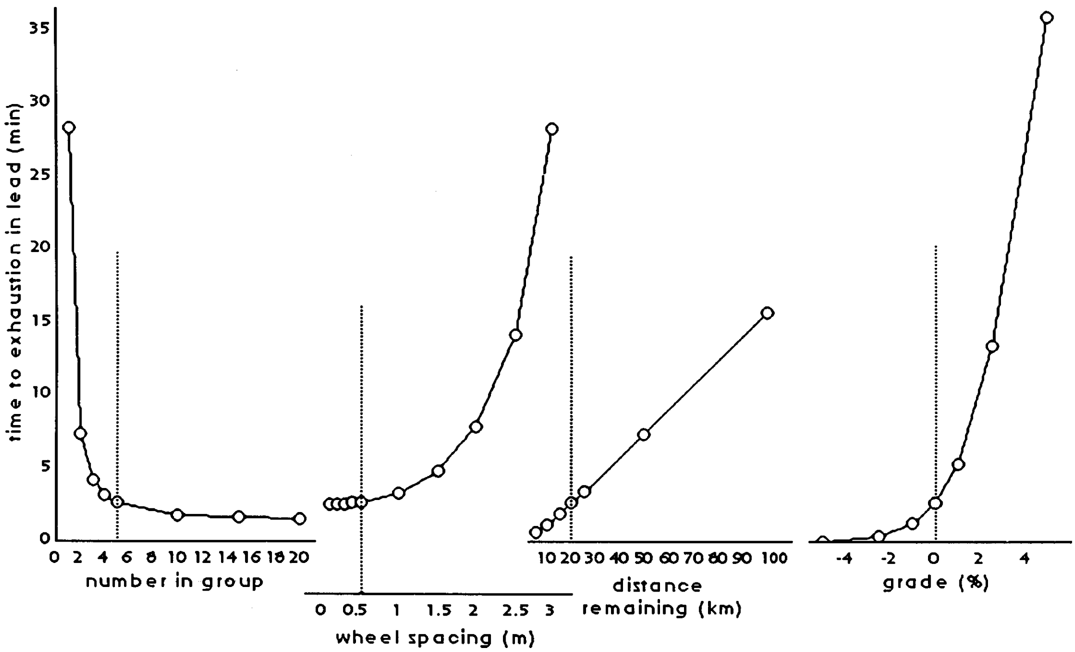
Fig. 4 The relationship between selected model variables and the mean velocity that a group of riders can sustain (*common ordinate*). The *dotted lines* indicate the baseline values for the simulations

falling more rapidly thereafter. Velocity also declines rapidly and approximately linearly as grade increases.

Figure 5 shows the effect of these variables on the time to exhaustion while leading, i.e. the maximum sustainable lead time for any rider. This tends to flatten out at about 2 min as the number in the group increases (large groups travel faster, so riders cannot tolerate longer lead times). It should be noted that this represents a maximum lead time, and does not take into account recovery kinetics. Often the required power outputs when leading are well above a putative "anaerobic threshold", and would be expected to result in rapid acidosis. Lead time increases rapidly as wheel spacing and grade increase. This is because under these conditions the group velocity becomes slower, the time required to finish the race increases and the sustainable fraction of $\dot{V}O_{2\,max}$ increases. Maximum lead time rises approximately linearly as the distance remaining increases.

Fig. 5 The relationship between selected model variables and the maximum lead time any rider in a group can sustain (*common ordinate*). The *dotted lines* indicate the baseline values for the simulations

Figure 6 shows the effect of the number of riders in the breakaway and chasing groups, grade and wheel spacing on the critical lead that a breakaway must have



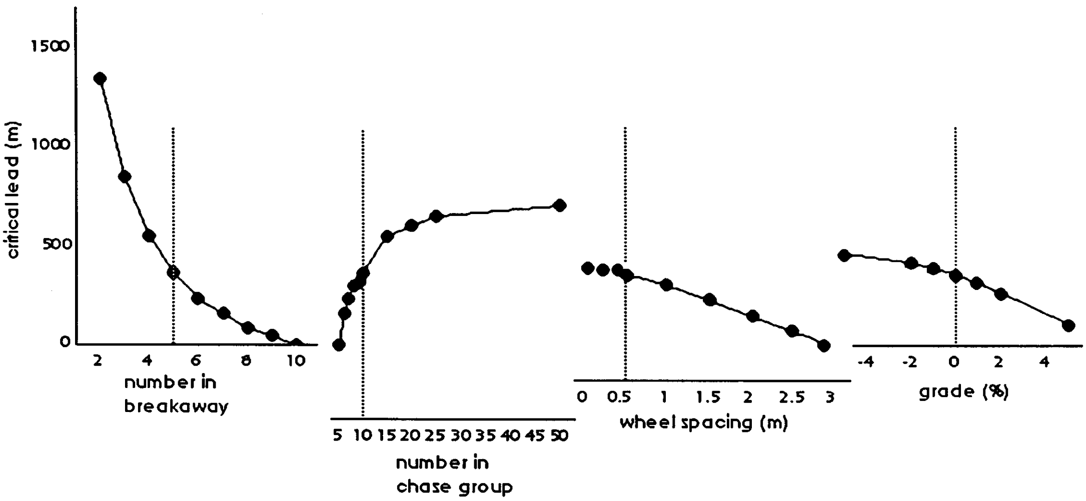


Fig. 6 The relationship between selected model variables and lead that a breakaway would require for success (*common ordinate*). The *dotted lines* indicate the baseline values for the simulations

for success. The required lead falls sharply as the number of riders in the breakaway increases, tending to flatten out as the number of riders in the breakaway approaches the number in the chasing group. As expected, the greater the number of riders in the chasing group, the greater the lead the breakaway requires for success. The curvilinear nature of this relationship shows that there are diminishing returns from larger chasing groups. The required lead becomes smaller as wheel spacing increases, and as grade increases.

Discussion

Kyle's (1979) drafting model described differences between individual and team 4000 m pursuit times quite well, and agreed moderately well with separate wind-tunnel data (Kyle and Burke 1984). McCole et al. (1990) measured the $\dot{V}O_2$ of cyclists riding behind lead vehicles and pacelines. These authors reported reductions of 18(11)%, 28(10)% and 26(8)% in $\dot{V}O_2$ for cyclists travelling at 32, 37 and 40 km $\cdot$ h^{-1} respectively with a 0.15–0.45 m wheel spacing. Using Eq. 1, the reduction in air resistance would be about 38%, which would result in estimated reductions in $\dot{V}O_2$ of 27%, 29–30% and 30–31% at the three speeds – results which are in reasonably good agreement with those recorded by McCole et al. (1990). These investigators also confirmed that there was no difference between drafting behind one, two or four riders in a paceline.

The preceding calculations apply for breakaway and chasing pacelines only (i.e. for riders aligned directly one behind the other). The effect of drafting is less if there is lateral displacement between the lead and following cyclists. Pugh (1970, 1971) reported this phenomenon in runners, where lateral displacements of 40–70 cm resulted in reductions in wind resistance of 4–80%, as opposed to 89–100% for runners directly behind the lead runner. Kyle (1979) reported drag reductions of 0–30% for laterally displaced riders as opposed to a 44% reduction for riders directly behind the lead cyclist. It is likely that lead riders leave a shielded "comet's tail" behind them, with reduction in air resistance diminishing as one moves backwards and laterally. CF_{draft} will also be affected by the formation of the group of riders. McCole et al. (1990) found a reduction of 62(6)% in $\dot{V}O_2$ when drafting behind a pace vehicle, equivalent to a 76% reduction in drag – about twice as great as drafting in a paceline. Behind a pack of eight riders, the reduction in $\dot{V}O_2$ was 39(6)%, equivalent to a 48% reduction in drag – 1.25 times as great as drafting in a paceline.

It has been suggested that riding close behind a leading cyclist will also assist the leading rider in that the low pressure area behind the cyclist will be "filled up" by the trailing rider. However, both Kyle (1979) and McCole et al. (1990) failed to find any measurable effect either in rolldown experiments or in field $\dot{V}O_2$ measurements.

These results regarding the effect of riding in groups are essentially similar to those of Kyle (1979). Differences between the values presented here and Kyle's values are probably due to slightly different assumptions about CF_{draft}, Kyle's failure to consider $^{ext}P_{roll}$, and Kyle's assumption that when dropping to the rear of the paceline, the rider is using the same power as when at the head of the paceline. This last assumption is odd, since at this point the rider is clearly using less power than the lead rider. The present model assumes that at this point the rider is using the same power as when drafting.

There are conditions under which the chasing group will never catch the breakaway. This will occur when the number in the chasing group is less than the number in the breakaway, and when wheel spacing is ≥ 3 m. The break that the chasing group can allow the breakaway to assume depends on the relative sizes of the chasing and breakaway groups (the larger the ratio of chasing to breakaway groups, the greater the permissible breakaway lead), how closely the groups draft (the closer the drafting, even if both groups draft at the same distance, the larger the permissible breakaway) and the slope of the terrain. The breakaway has the best chance of success when it has to defend its lead on an uphill slope. The reason for this is that under these conditions the proportion of total external power output devoted to overcoming air resistance is reduced. It is the ability of the chasing group to reduce overall air resistance by more than the breakaway group, in virtue of the larger numbers in the chasing group, that constitutes the critical advantage of the chasing group. For the same reason, any conditions which tend to reduce the relative importance of the power required to overcome air resistance (e.g. rough roads, still conditions, a slower overall race pace) will favour the breakaway, while factors working in the opposite direction (smooth roads, headwinds, a faster race pace) will favour the chasing group. Changes in the $\dot{V}O_{2\,max}$ of the riders have little or no effect on the critical lead. As the distance remaining increases, the critical lead will naturally increase, but remains very nearly a constant fraction of the distance remaining.

While Fig. 6 shows the general pattern of the relationship between different variables and breakaway success, it will vary according to the chosen baseline values, and according to how the breakaway performance is expressed. It is possible, for example, that while the ratio of breakaway speed to chasing group speed may increase as the value of one variable is changed, the critical lead may also increase. This can occur when the effect of a change in a variable is to slow down the speed of both groups.

The validity of the foregoing analysis is contingent upon the following assumptions:

1. The groups consist of identical riders (i.e. with identical values for supply and demand variables).
2. They share the lead evenly.
3. They are neither accelerating nor decelerating, so that the power required to change the kinetic energy of the system is zero.
4. They are riding frictionless bicycles.
5. They maintain the same power output that they would if riding alone.

Naturally, these assumptions are not met in practice. It is unlikely that riders can maintain the same power output whatever the pattern of power production, i.e. the relationship between the sustainable fraction of $\dot{V}O_{2\,max}$ (or mean power) and time to exhaustion will be different with the "burst" pattern of power production required when rotating the lead, than in the "steady-state" pattern when riding alone. The metabolic effects of intermittent exercise are quite different from those of continuous constant-power exercise. In order to adequately model the effects of group riding, we must be able to model recovery kinetics. In practice, riders rarely share the lead evenly, particularly as the breakaway group nears the finish and "cat and mouse" tactics come into play. Nonetheless, it remains to be seen to what extent the violation of these assumptions will affect the predictive power of the model.

An interesting question, and a challenging one for further research, is the optimal distribution of lead times and speeds in a group of riders with different physiological capacities and equipment. This is a very complex optimization problem, but one of great practical importance in events such as the team pursuit. However, a solution will depend on an accurate model of fatigue and recovery kinetics.

Acknowledgements The author would like to acknowledge the assistance of Dr. Kevin Norton in the development of the mathematical model on which this paper is based. This study was supported in part by a grant from the Australian Sports Commission.

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